INTEGRAL 2

Matematika

FTP – UB

Latihan

- Kerjakan !

$$\int \cos^4 x \, dx$$

$$\int \frac{dx}{\sin^2 2x \cos^4 2x}$$

$$\int \frac{x^2}{(2+3x)^{2/3}} \, dx$$

$$\int \frac{\sqrt{9-4x^2}}{x} \, dx$$

- Bagaimana jika integral dibatasi mulai 1 sampai 10?

Contoh soal

$$\int \cos^4 x \, dx = ...$$

Jawabannya:

$$\int cos^4 x \, dx = \int \left[\frac{1}{2} \left(1 + cos\, 2x \right) \right]^2 dx = \frac{1}{4} \int \left[1 + 2 cos\, 2x + cos^2 2x \right] dx$$

$$= \frac{1}{4} \left[\frac{3}{2} x + sin\, 2x + \frac{1}{8} sin\, 4x \right] + c$$

Contoh soal

$$\int \frac{dx}{\sin^2 2x \cos^4 2x} = \int \operatorname{cosec}^2 2x \sec^4 2x \, dx = \int \operatorname{cosec}^2 2x \sec^2 2x \sec^2 2x \, dx$$

$$= \frac{1}{2} \int [1 + \cot g^2 2x][1 + tg^2 2x] \, d(tg\, 2x)$$

$$= \frac{1}{2} \int \left[1 + \frac{1}{tg^2 2x}\right][1 + tg^2 2x] \, d(tg\, 2x)$$

$$= \frac{1}{2} \int \left[2 + \frac{1}{tg^2 2x} + tg^2 2x\right] d(tg\, 2x)$$

$$= \frac{1}{2}\left(2tg\, 2x - \frac{1}{tg\, 2x} + \frac{1}{3} tg^3 2x\right) + c$$

Contoh soal

$$\int \frac{x^2}{(2+3x)^{2/3}}\,dx = \ldots\ldots$$

substitusi $u^3 = 2+3x$ $\quad x = \frac{1}{3}(u^3 - 2)$

$$d(u^3) = d(2+3x)$$

$$3u^2\,du = 3dx$$

$$dx = u^2\,du$$

Sehingga

$$\int \frac{x^2}{(2+3x)^{2/3}}dx = \int \frac{\left[\frac{1}{3}\left(u^3-2\right)\right]^2}{\left[u^3\right]^{2/3}}u^2du = \frac{1}{9}\int \frac{\left(u^3-2\right)^2}{u^2}u^2du$$

$$= \frac{1}{9}\int \left[u^6-4u^3+4\right]du$$

$$= \frac{1}{9}\left[\frac{1}{7}u^7-u^4+4u\right]+c$$

$$= \frac{1}{9}\left[\frac{1}{7}(2+3x)^{7/3}-(2+3x)^{4/3}+4(2+3x)^{1/3}\right]+c$$

$$= \frac{1}{63}(2+3x)^{1/3}\left[(2+3x)^2-7(2+3x)+28\right]+c$$

contoh soal

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = ...$$

jawab :

$$x = \frac{3}{2}\sin z \quad \rightarrow \quad dx = \frac{3}{2}\cos z\,dz \qquad \sqrt{9-4x^2} = 3\cos z$$

,

Jadi,

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \int \frac{3\cos z}{\frac{3}{2}\sin z}\left(\frac{3}{2}\cos z\,dz\right) = 3\int \frac{\cos^2 z}{\sin z}\,dz$$

$$3\int \frac{1-\sin^2 z}{\sin z}dz = \quad 3\int \cos ec\, z\,dz - 3\int \sin z\,dz$$

$$= 3\ln|\operatorname{cosec} z - \cot z| + 3\cos z + c$$

$$= 3\ln\left|\frac{3-\sqrt{9-4x^2}}{2x}\right| + \sqrt{9-4x^2} + c$$

Integral Tertentu

- Integral tertentu adalah integral dari suatu fungsi yang nilai-nilai variabel bebasnya (memiliki batas-batas) tertentu.
- Jika fungsi terdefinisi pada interval tertutup [a,b] , maka integral tertentu dari a ke b dinyatakan oleh :

$$\int_a^b f(x)dx$$

- Dimana :

  f(x) : integran

  a : batas bawah

  b : batas atas

Perbedaan Intergral

- Integral tak tentu hasilnya berupa fungsi sedangkan integral tertentu (integral Riemann) hasilnya berupa bilangan.

KAIDAH-KAIDAH INTEGRASI TERTENTU

$$\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

$$\int_2^5 x^4 dx = \left[\frac{x^5}{5}\right]_2^5 = \frac{1}{5}\left[x^5\right]_2^5 = \frac{1}{5}\left(5^5 - 2^5\right)$$

$$= \frac{1}{5}(3125 - 32) = 618,6$$

$$\int_a^a f(x)dx = 0$$

$$\int_2^2 x^4 dx = \left[\frac{x^5}{5}\right]_2^2 = \frac{1}{5}\left[x^5\right]_2^2 = \frac{1}{5}\left(2^5 - 2^5\right)$$

$$= \frac{1}{5}(32 - 32) = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$-\int_5^2 x^4 dx = -\left[\frac{x^5}{5}\right]_5^2 = -\frac{1}{5}\left[x^5\right]_5^2 = -\frac{1}{5}\left(2^5 - 5^5\right)$$

$$= -\frac{1}{5}(32 - 3125) = 618,6$$

KAIDAH-KAIDAH INTEGRASI TERTENTU

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

$$\int_2^5 5x^4 dx = 5\left[\frac{x^5}{5}\right]_2^5 = 5.\frac{1}{5}\left[x^5\right]_2^5$$

$$= 3125 - 32 = 3093$$

$$\int_a^b \{f(x) + g(x)\}dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_2^5 \left(x^4 + 5x^4\right)dx = \int_2^5 x^4 dx + \int_2^5 5x^4 dx$$

$$= 618{,}6 + 3093 = 3711{,}6$$

$$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$$

$$\int_2^3 x^4 dx + \int_3^5 x^4 dx = \int_2^5 x^4 dx = 618{,}6$$