Pengantar Aljabar

Matematika Industri I

TIP – FTP – UB

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Pokok Bahasan

- **Pernyataan aljabar**
- **Pangkat**
- **Logaritma**
- **Perkalian pernyataan aljabar suatu variabel tunggal**
- **Pecahan**
- **Pembagian satu pernyataan dengan pernyataan lain**
- **Faktorisasi pernyataan aljabar**

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Algebraic expressions

Symbols other than numerals

Constants

Variables

Rules of algebra

Rules of precedence

Terms and coefficients

Collecting like terms

Similar terms

Expanding brackets

Nested brackets

Algebraic expressions

Symbols other than numerals

An unknown number can be represented by a letter of the alphabet which can then be manipulated just like an ordinary numeral within an arithmetic expression. Manipulating letters and numerals within arithmetic expressions is referred to as *algebra*.

Algebraic expressions

Constants and variables

Sometimes a letter represents a single number. Such a letter is referred to as a *constant*. Other times a letter may represent one of a collection of numbers. Such a letter is referred to as a *variable*.

Algebraic expressions

Rules of algebra

The rules of arithmetic, when expressed in general terms using letters of the alphabet are referred to as the rules of algebra. Amongst these rules are those that deal with:

Commutativity
Associativity
Distributivity

Algebraic expressions

Rules of algebra

Commutativity

Both addition and multiplication are commutative operations. That is, they can be added or multiplied in any order without affecting the result:

$$x+y=y+x \text{ and } xy=yx$$

Note that the multiplication sign is suppressed:

$$x\times y \text{ is written as } xy$$

Algebraic expressions

Rules of algebra

Associativity

Both addition and multiplication are associative operations. That is, they can be associated in any order without affecting the result:

$$x+(y+z)=(x+y)+z=x+y+z$$

$$x(yz)=(xy)z=xyz$$

Algebraic expressions

Rules of algebra

Distributivity

Multiplication is distributive over addition and subtraction from both the left and the right:

$$x(y+z)=xy+xz \text{ and } x(y-z)=xy-xz$$

$$(x+y)z=xz+yz \text{ and } (x-y)z=xz-yz$$

Algebraic expressions

Rules of algebra

Distributivity

Division is distributive over addition and subtraction from the right but not from the left:

$$(x+y)\div z = x\div z + y\div z \text{ and } (x-y)\div z = x\div z - y\div z$$

$$x\div(y+z) \neq x\div y + x\div z \text{ and } x\div(y-z) \neq x\div y - x\div z$$

Algebraic expressions

Rules of precedence

The familiar rules of precedence continue to apply when algebraic expressions involving mixed operations are to be manipulated

Algebraic expressions

Terms and coefficients

An algebraic expression consists of alphabetic characters and numerals linked together with the arithmetic operations. For example:

$$8x - 3xy$$

Each component of this expression is called a *term* of the expression. Here there are two terms, namely the x term and the xy term.

The numbers 8 and –3 are called the *coefficients* of their respective terms.

Algebraic expressions

Collecting like terms

Terms that have the same variables are called *like* terms and like terms can be collected together by addition and subtraction. In this manner, expressions can be simplified.

Algebraic expressions

Similar terms

Terms that have variables in common are called *similar* terms and similar terms can be collected together by *factorization*. The symbols the terms have in common are called *common factors*. For example:

$$ab+bc=b(a+c)$$

Here, b is a common factor that has been factorized out by the introduction of *brackets*.

Algebraic expressions

Expanding brackets

Sometimes it is desired to reverse the process of factorizing an expression by *removing* the brackets (called *expanding the brackets*). This is done by:

(a) multiplying or dividing each term inside the bracket by the term outside the bracket, but

(b) If the term outside the bracket is negative then each term inside the bracket changes sign

Algebraic expressions

Nested brackets

In expanding brackets where an algebraic expression contains brackets nested within other brackets the innermost brackets are removed first.

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Powers

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Rules of indices

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The use of *powers* in the first instance (also called *indices* or *exponents*) provides a convenient form of algebraic shorthand for repetitive multiplication.

Powers

Rules of indices

Three basic rules are:

$$1 \quad a^m \times a^n = a^{m+n}$$

$$2 \quad a^m \div a^n = a^{m-n}$$

$$3 \quad \left(a^m\right)^n = a^{mn}$$

These lead to:

$$4 \quad a^0 = 1$$

$$5 \quad a^{-m} = \frac{1}{a^m}$$

$$6 \quad a^{\frac{1}{m}} = \sqrt[m]{a}$$

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Logarithms

Powers

Logarithms

Rules of logarithms

Base 10 and base e

Change of base

Logarithmic equations

Logarithms

Powers

Any real number can be written as another number written raised to a power. For example:

$$9 = 3^2 \text{ and } 27 = 3^3$$

So that:

$$9 \times 27 = 3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$$

Here the process of *multiplication* is replaced by the process of relating numbers to powers and then *adding* the powers – a simpler operation.

Logarithms

Logarithms

If a, b and c are three real numbers where:

$$a = b^c \text{ and } b > 1$$

The power c is called the logarithm of the number a to the base b and is written:

$$c = \log_b a \text{ spoken as } c \textit{ is the log of } a \textit{ to the base } b$$

Logarithms

Rules of logarithms

The three basic rules are:

$$(a)\quad \log_a xy = \log_a x + \log_a y$$
$$(b)\quad \log_a x \div y = \log_a x - \log_a y$$
$$(c)\quad \log_a x^n = n\log_a x$$

These lead to:

$$(d)\quad \log_a a = 1$$
$$(e)\quad \log_a a^x = x$$
$$(f)\quad a^{\log_a x} = x$$
$$(g)\quad \log_a b = 1/\log_b a$$

Logarithms

Base 10 and base e

On a calculator there are buttons that provide access to logarithms to two different bases, namely 10 and the exponential number $e = 2.71828 \ldots$

Logarithms to base 10 are called common logarithms and are written without indicating the base as **log**

Logarithms to base e are called natural logarithms and are written as **ln**

Logarithms

Change of base

The change of base formula that relates the logarithms of a number to two different bases is given as:

$$\log_b a \times \log_a x = \log_b x$$

Logarithms

Logarithmic equations

Logarithmic expressions and equations can be manipulated using the rules of logarithms. Example:

$$
\begin{aligned}
\log_a x^2 + 3\log_a x - 2\log_a 4x \\
&= \log_a x^2 + \log_a x^3 - \log_a (4x)^2 \\
&= \log_a \left(\frac{x^2 x^3}{16x^2} \right) \\
&= \log_a \left(\frac{x^3}{16} \right)
\end{aligned}
$$

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Multiplication of algebraic expressions of a single variable

Brackets are multiplied out a term at a time. For example:

$$\begin{aligned}(2x+5)(x^2+3x+4)&\\ &=2x(x^2+3x+4)+5(x^2+3x+4)\\ &=2x^3+6x^2+8x+5x^2+15x+20\\ &=2x^3+11x^2+23x+20\end{aligned}$$

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Fractions

Algebraic fractions

Addition and subtraction

Multiplication and division

Fractions

Algebraic fractions

A numerical fraction is represented by one integer divided by another. Division of symbols follows the same rules to create *algebraic fractions*. For example,

$$5 \div 3 \text{ can be written as } \frac{5}{3} \text{ so } a \div b \text{ can be written as } \frac{a}{b}$$

Fractions

Addition and subtraction

The addition and subtraction of algebraic fractions follow the same rules as the addition and subtraction of numerical fractions – the operations can only be performed when the denominators are the same.

Fractions

Multiplication and division

Just like numerical fractions, algebraic fractions are multiplied by multiplying their numerators and denominators separately.

To divide by an algebraic fraction multiply by its *reciprocal*.

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Division of one expression by another

Division is defined as repetitive subtraction and we set out the division of one expression by another in the same way as we set out the long division of two numbers. For example:

$$
\begin{array}{r@{}l}
 & 4x^2-6x+7 \\
3x+4 & \overline{)\,12x^3-2x^2-3x+28} \\
 & \underline{\,12x^3+16x^2\,} \\
 & -18x^2-3x \\
 & \underline{-18x^2-24x} \\
 & 21x+28 \\
 & \underline{21x+28} \\
 & \bullet\quad\bullet
\end{array}
$$

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Factorization of algebraic expressions

Common factors

Common factors by grouping

Useful products of two simple factors

Quadratic expressions as the product of two simple factors

Factorization of a quadratic expression

Test for simple factors

Factorization of algebraic expressions

Common factors

The simplest form of factorization is the extraction of highest common factors from a pair of expressions. For example:

$$35x^2y^2 - 10xy^3 = 5xy^2(7x - 2y)$$

Factorization of algebraic expressions

Common factors by grouping

Four termed expressions can sometimes be factorized by grouping into two binomial expressions and extracting common factors from each. For example:

$$
\begin{aligned}
&2ac+6bc+ad+3bd \\
&\qquad = (2ac+6bc)+(ad+3bd) \\
&\qquad = 2c(a+3b)+d(a+3b) \\
&\qquad = (a+3b)(2c+d)
\end{aligned}
$$

Factorization of algebraic expressions

Useful products of two simple factors

A number of standard results are worth remembering:

$$(a)\quad (a+b)^2 = a^2 + 2ab + b^2$$

$$(b)\quad (a-b)^2 = a^2 - 2ab + b^2$$

$$(c)\quad (a-b)(a+b) = a^2 - b^2$$

Factorization of algebraic expressions

Quadratic expressions as the product of two simple factors

$$(a) \quad (x+g)(x+k) = x^2 + (g+k)x + gk$$

$$(b) \quad (x-g)(x-k) = x^2 - (g+k)x + gk$$

$$(c) \quad (x+g)(x-k) = x^2 + (g-k)x - gk$$

Factorization of algebraic expressions

Factorization of a quadratic expression $ax^2 + bx + c$ when $a = 1$

The factorization is given as:

$$x^2 + bx + c = (x + f_1)(x + f_2)$$

Where, if $c > 0$, f_1 and f_2 are factors of c whose sum equals b, both factors having the same sign as b.

If $c < 0$, f_1 and f_2 are factors of c with opposite signs, the numerically larger having the same sign as b and their difference being equal to b.

Factorization of algebraic expressions

Factorization of a quadratic expression $ax^2 + bx + c$ *when* $a \neq 1$

The factorization is given as:

$$ax^2 + bx + c = ax^2 + f_1 x + f_2 x + c$$

Where, if $c > 0$, f_1 and f_2 are two factors of $|ac|$ whose sum equals $|b|$, both factors having the same sign as b.

If $c < 0$ their values differ by the value of $|b|$, the numerically larger of the two having the same sign as b and the other factor having the opposite sign.

Factorization of algebraic expressions

Test for simple factors

The quadratic expression:

$$ax^2 + bx + c$$

Has simple factors if, and only if:

$$b^2 - 4ac = k^2 \text{ for some integer } k$$

Hasil Pembelajaran

- Menggunakan simbol alfabet untuk melengkapi numeral dan mengkombinasikan simbol dengan menggunakan operasi aritmetika
- Menyederhanakan pernyataan aljabar
- Menghilangkan tanda kurung sehingga mendapatkan pernyataan aljabar alternatif
- Memanipulasi pernyataan dengan pangkat
- Memanipulasi logaritma
- Memanipulasi pecahan
- Memfaktorisasi pernyataan aljabar