DIFFERENTIATION

Standard derivatives

Functions of a function

Logarithmic differentiation

Implicit functions

Parametric equations

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Standard derivatives

As an aide memoir this slide and the following slide contain the list of standard derivatives which should be familiar.

$y = f(x)$	dy/dx
x^n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
a^x	$a^x.\ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x.\ln a}$

Standard derivatives

$y = f(x)$	dy/dx
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\text{cosec} x$	$-\text{cosec} x.\cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$

Functions of a function

If: $y = f(u)$ and $u = F(x)$

then

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$ (called the 'chain rule')

For example: $y = \cos(5x - 4)$ so $y = \cos u$ and $u = 5x - 4$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

$$= (-\sin u).5$$

$$= -5\sin(5x - 4)$$

Functions of a function

If: $y = \ln u$ and $u = F(x)$

then

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{1}{F}.\frac{dF}{dx}$$

For example: $y = \ln \sin x$ then

$$\frac{dy}{dx} = \frac{1}{\sin x}.\cos x$$
$$= \cot x$$

Functions of a function

Products

Quotients

Functions of a function

Products

If: $y = uv$

then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

For example: $y = x^3.\sin 3x$ then

$$\frac{dy}{dx} = x^3.3\cos 3x + 3x^2 \sin 3x$$
$$= 3x^2\left(x\cos 3x + \sin 3x\right)$$

Functions of a function

Quotients

If: $y = \dfrac{u}{v}$

then

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

For example: $y = \dfrac{\sin 3x}{x+1}$ then

$$\frac{dy}{dx} = \frac{(x+1)3\cos 3x - \sin 3x.1}{(x+1)^2}$$

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Logarithmic differentiation

Taking logs before differentiating may simplify the working. For example:

$$y = \frac{x^2 \sin x}{\cos 2x} \quad \text{then} \quad \ln y = \ln\left(x^2\right) + \ln(\sin x) - \ln(\cos 2x)$$

$$\frac{d \ln y}{dx} = \frac{x^2}{2x} + \frac{\cos x}{\sin x} - \frac{-2\sin 2x}{\cos 2x} = \frac{x}{2} + \cot x + 2\tan 2x = \frac{1}{y}.\frac{dy}{dx}$$

Therefore:

$$\frac{dy}{dx} = \frac{x^2 \sin x}{\cos 2x}\left(\frac{x}{2} + \cot x + 2\tan 2x\right)$$

Implicit functions

There are times when the relationship between x and y is more involved and y cannot be simply found in terms of x. For example:

$$xy + \sin y = 2$$

In this case a relationship of the form $y = f(x)$ is *implied* in the given equation.

Implicit functions

To differentiate an implicit function remember that *y* is a function of *x*. For example, given:

$$x^2 + y^2 = 25$$

Differentiating throughout gives:

$$2x + 2y\frac{dy}{dx} = 0$$

so.

$$\boxed{\frac{dy}{dx} = \frac{x}{y}}$$

Standard derivatives

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Parametric equations

Sometimes the relationship between x and y is given via a third variable called a *parameter*. For example, from the two *parametric equations*

$$x = \sin t \quad \text{and} \quad y = \cos 2t$$

the derivative of *y* with respect to *x* is found as follows:

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -2\sin 2t.\frac{1}{\cos t} \quad \left(\text{since } \frac{dt}{dx} = \frac{1}{dx/dt}\right)$$

$$= -4\sin t \cos t.\frac{1}{\cos t}$$

$$= -4\sin t$$

Parametric equations

The second derivative is found as follows:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}.\frac{dy}{dx} = \frac{d}{dx}.\left(-4\sin t\right)$$

$$= \frac{d}{dt}.\left(-4\sin t\right).\frac{dt}{dx}$$

$$= -4\cos t.\frac{1}{\cos t}$$

$$= -4$$

Learning outcomes

- ✓Differentiate by using a list of standard derivatives
- ✓Apply the chain rule
- ✓Apply the product and quotient rules
- ✓Perform logarithmic differentiation
- ✓Differentiate implicit functions
- ✓Differentiate parametric equations