DIFFERENTIATION APPLICATIONS 1

Equation of a straight line

Tangents and normals to a curve at a given point

Curvature

Centre of curvature

Differentiation applications

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Equation of a straight line

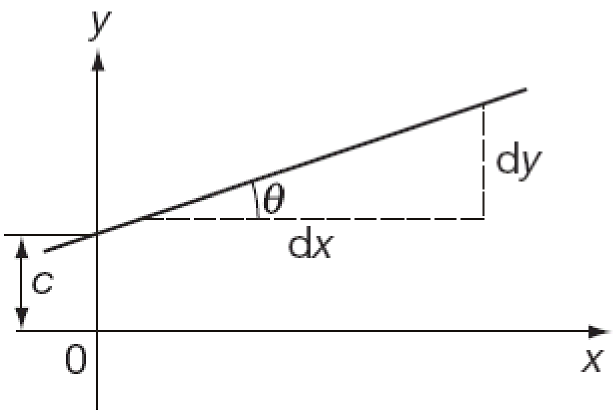
The basic equation of a straight line is:

$$y = mx + c$$

where:

$$m = \text{gradient } = \frac{dy}{dx}$$

$c =$ intercept on the y-axis



Equation of a straight line

Given the gradient m of a straight line and one point (x_1, y_1) through which it passes, the equation can be used in the form:

$$y - y_1 = m(x - x_1)$$

Equation of a straight line

If the gradient of a straight line is m and the gradient of a second straight line is m_1 where the two lines are mutually perpendicular then:

$$mm_1 = -1 \text{ that is } m_1 = -\frac{1}{m}$$

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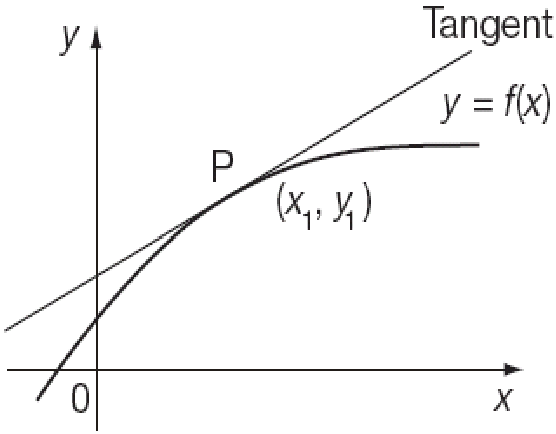
Tangent

The gradient of a curve, $y = f(x)$, at a point *P* with coordinates (x_1, y_1) is given by the derivative of *y* (the gradient of the tangent) at the point:

$$\frac{dy}{dx} \text{ at } (x_1, y_1)$$



The equation of the tangent can then be found from the equation:

$$y - y_1 = m(x - x_1) \text{ where } m = \frac{dy}{dx}$$

Tangents and normals to a curve at a given point

Normal

The gradient of a curve, $y = f(x)$, at a point P with coordinates (x_1, y_1) is given by the derivative of y (the gradient of the tangent) at the point:

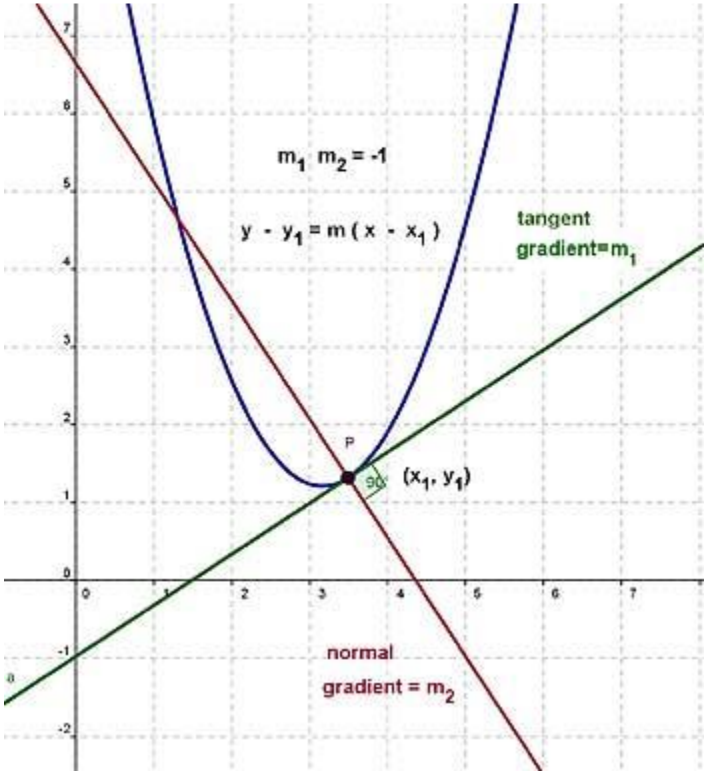
$$\frac{dy}{dx} \text{ at } (x_1, y_1)$$

The equation of the normal (*perpendicular to the tangent*) can then be found from the equation:

$$y - y_1 = m(x - x_1) \text{ where } m = -\frac{1}{dy/dx}$$

Tangents and normals to a curve at a given point

Tangent and normal



Source: www.a-levelmathstutor.com

Equation of a straight line

Tangents and normals to a curve at a given point

Curvature

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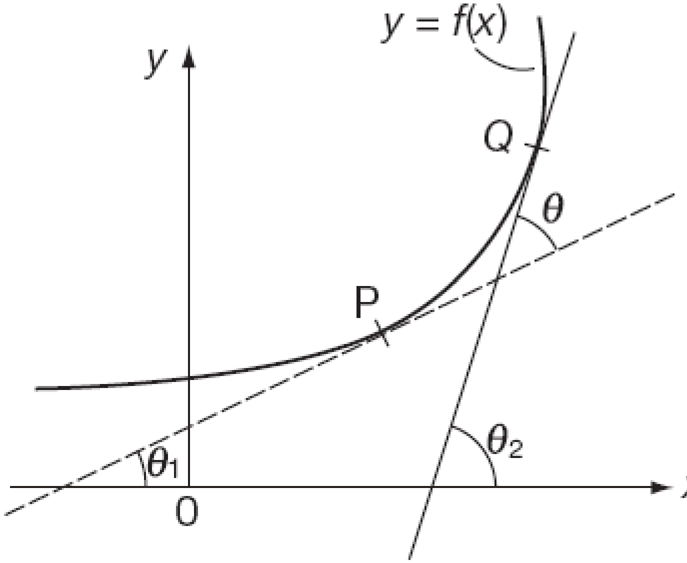
Curvature

The *curvature* of a curve at a point on the curve is concerned with how quickly the curve is changing direction in the neighbourhood of that point.

Given the gradients of the curve at two adjacent points P and Q it is possible to calculate the *change in direction* $\theta = \theta_2 - \theta_1$

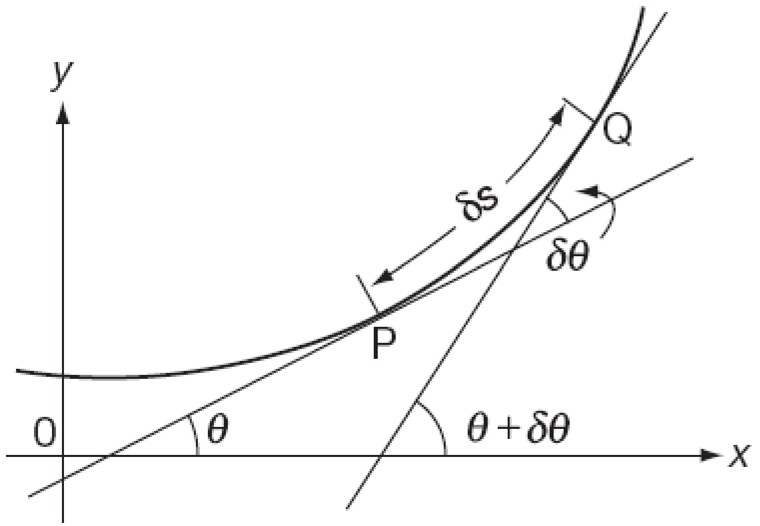


Curvature

The *distance* over which this change takes place is given by the arc PQ.

For small changes in direction $\delta\theta$ and small arc distance δs the average rate of change of direction with respect to arc length is then:

$$\frac{\delta\theta}{\delta s}$$



Curvature

A small arc PQ approximates to the arc of a circle of radius R where:

$$\text{arcPQ} = \delta s \cong R\delta\theta$$

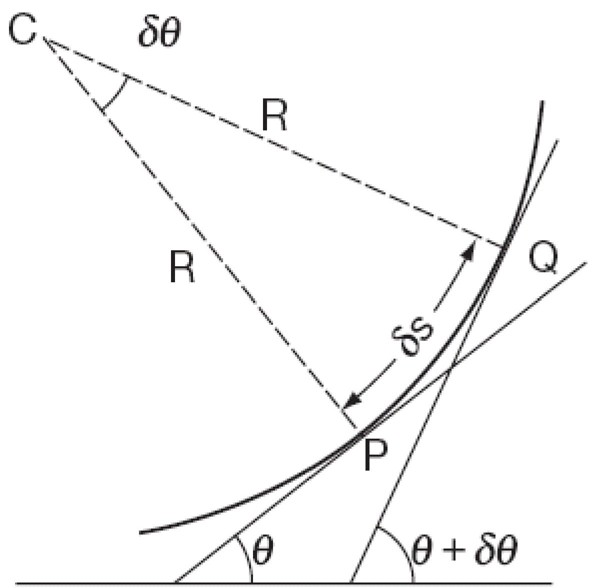
So

$$\frac{\delta\theta}{\delta s} \cong \frac{1}{R}$$

and in the limit as

$$\delta s \to 0 \;\; \text{this becomes} \;\; \frac{d\theta}{ds} = \frac{1}{R}$$

Which is the curvature at P; R being the *radius of curvature*



Curvature

The radius of curvature R can be shown to be given by:

$$R = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

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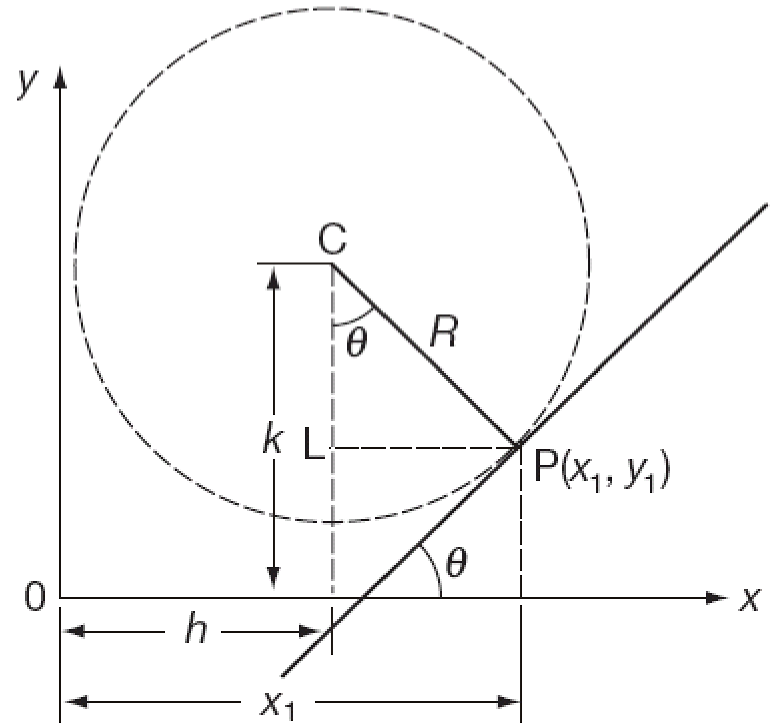
If the centre of curvature *C* is located at the point (*h*, *k*) then:



$$h = x_1 - LP = x_1 - R\sin\theta$$
$$k = y_1 + LC = y_1 + R\cos\theta$$

Learning outcomes

- ✓Evaluate the gradient of a straight line
- ✓ Recognize the relationship satisfied by two mutually perpendicular lines
- ✓Derive the equations of a tangent and a normal to a curve
- ✓Evaluate the curvature and radius of curvature at a point on a curve
- ✓Locate the centre of curvature for a point on a curve